Date: 10/4/18

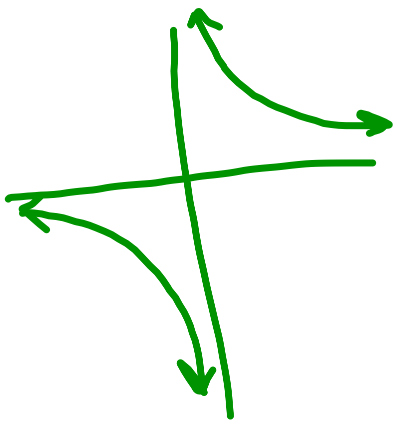
Chp: Chp. 2:2 → Limits involving ∞

Obj: • Be able to work w/ finite & infinite limits
• Describe end behavior

* ∞ does not a real #
* ∞ represents a direction
* $\lim\limits_{x \to \infty}$ means as x moves increasingly right on a # line
* $\lim\limits_{x \to -\infty}$ same as right example but left
* Limits approaching -/+ ∞ may or may not exist.

Ex. 1

$f(x) = \frac{1}{x}$



a) $\lim_{x \to \infty} f(x) = 0$

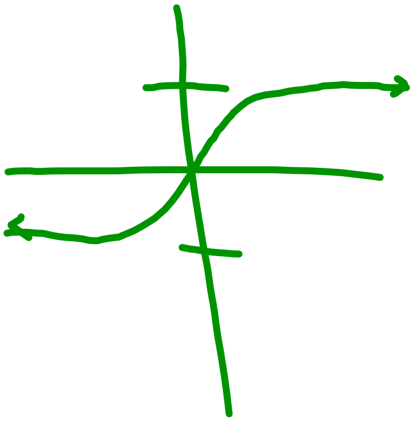
b) $\lim_{x \to -\infty} f(x) = 0$

* Horizontal Asymptote = The line $y = b$ is a horizontal asymptote of the graph of the function $y = f(x)$ if either

$\lim_{x \to \infty} f(x) = b$ or $\lim_{x \to -\infty} f(x) = b$

Ex. 2

$$f(x) = \frac{x}{\sqrt{x^2+1}}$$



a) $\lim_{x \to \infty} f(x) = 1$

b) $\lim_{x \to -\infty} f(x) = -1$

Ex. 3

$$\lim_{x \to \infty} \frac{\sin x}{x} = 0$$

* Properties of limits apply to infinite limits.

Ex. 4

$$\lim_{x \to \infty} \frac{5x + \sin x}{x} = 5$$

$$\frac{5\cancel{x}}{\cancel{x}} + \frac{\sin x}{x}$$

$$5 + 0 = 5$$

If the values of $f(x)$ outgrow all bounds as $x \to$ a finite #,

$$\lim_{x \to a} f(x) = \infty \text{ or } -\infty$$

Ex 5. $f(x) = \frac{1}{x}$

$\lim_{x \to 0^+} f(x) = \infty$

$\lim_{x \to 0^-} f(x) = -\infty$

$\therefore$ 0 is a vertical asymtote

Formal Definition: The line $x = a$ is a vertical asymtote of the graph of the function $y = f(x)$ if either:

$$\lim_{x \to a^+} f(x) = \pm\infty \text{ or } \lim_{x \to a^-} f(x) = \pm\infty$$

Ex 6. Find VA of $f(x) = \frac{1}{x^2}$

Describe the behavior

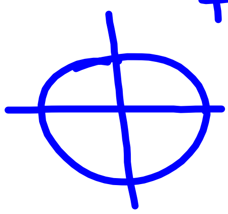
$$\lim_{x \to 0^+} \frac{1}{x^2} = \infty$$

$$\lim_{x \to 0^-} \frac{1}{x^2} = \infty$$

VA $x = 0$

Ex 7 Find VA of $f(x) = \tan x$

$$f(x) = \frac{\sin x}{\cos x}$$



$$\cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$$

$x = \frac{k\pi}{2}$ where k is odd integer

Just because the denominator = 0, does not mean there is a VA @ that spot.

Ex 8 $f(x) = \frac{\sin x}{x}$

$\lim_{x \to 0} \frac{\sin x}{x} = 1$ not $\pm\infty$

End Behavior Models

As $|x|$ becomes very large ($|x| \to \infty$) we can sometimes model the behavior of a complicated function by a simpler one.

Ex 9. $f(x) = 3x^4 - 2x^3 + 3x^2 - 5x + 6$

$g(x) = 3x^4$

Graphs of $f(x)$ and $g(x)$ are different at the origin but virtually identical for $|x| \to \infty$

Formal Definition: $g(x)$ is

right end behavior model for $f(x)$ if & only if

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = 1$$

left end behavior model for $f(x)$ ...

$$\lim_{x\to-\infty} \frac{f(x)}{g(x)} = 1$$

if it works for both $\Rightarrow$ end behavior model (EBM)

Ex 10

$$f(x) = \frac{2x^5 + x^4 - x^2 + 1}{3x^2 - 5x + 7}$$

EBM num. $2x^5$

EBM den. $3x^2$

EBM $f(x) = \frac{2x^5}{3x^2} = \frac{2x^3}{3}$

★ only look at 1^{st} term when $|x| \to \infty$

Trig Limits

3 Special Limits:

1. $\lim_{x \to 0} \frac{\sin x}{x} = 1$
2. $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$
3. $\lim_{x \to 0} \frac{\tan x}{x} = 1$

Limit rule applies for the reciprocal as well

i.e., $\lim_{x \to 0} \frac{\theta}{\sin\theta} = 1$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$

Reciprocal Identities: $\csc x = \frac{1}{\sin x}$

$\sec x = \frac{1}{\cos x}$

Properties of Complex Fractions:

Multiply by reciprocal

Unit Circle

③ $\lim_{x\to 0} \frac{\sin x}{3x} = \lim_{x\to 0}\left(\frac{\sin x}{x\cdot 3}\right) = 1\cdot\frac{1}{3} = \frac{1}{3}$

④ $\lim_{x\to 0} \frac{\cancel{\sin 5x}}{\cancel{\sin 3x}}\cdot\frac{5x}{\cancel{5x}}\cdot\frac{\cancel{3x}}{3x} = \lim_{x\to 0}\frac{5\cancel{x}}{3\cancel{x}} = \frac{5}{3}$

⑨ $\lim_{x\to 0} \frac{\cancel{\tan 2x}}{3\cancel{x}}\cdot\frac{2}{\cancel{2}} = \frac{2}{3}$

$$\lim_{x \to 0} \frac{5x^3+8x^2}{3x^4-16x^2}$$

$$\frac{\cancel{x^2}(5x+8)}{\cancel{x^2}(3x^2-16)} = \frac{8}{-16} = -\frac{1}{2}$$